

Household Finance Problems and Numerical Methods



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How do we optimally manage our Finances?



• Housing and Mortgages



• Health and Insurance



• Labor and Education

• Money and Investments



• Money and Investments



When the market is std. B/S:

$$dB(t) = rB(t)dt$$

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t)$$

Optimizing Expected Utility of Terminal Wealth:

Determine



$$V(t, x) = \sup_{\pi} E_{t,x}[u(X_T^{\pi})]$$

Non-stochastic



HJB-equation to solve:

$$V_t + \inf_{\pi} \{ (r + \pi(\alpha - r))xV_x + \frac{1}{2}\sigma^2\pi^2x^2V_{xx} \} = 0$$

$$V(T, x) = u(x)$$

In conclusion: many simple Household Finance Problems can be solved analytically by the HJB-equation

However, limitations often occur, e.g.:

- Controls continuous in time
- Time-homogenous utility function
- Conditions on smoothness and growth
- Complete markets

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manage our Fin



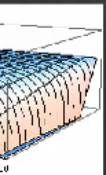
- Health and



- Housing and Mortgages

form solutions,
of the results

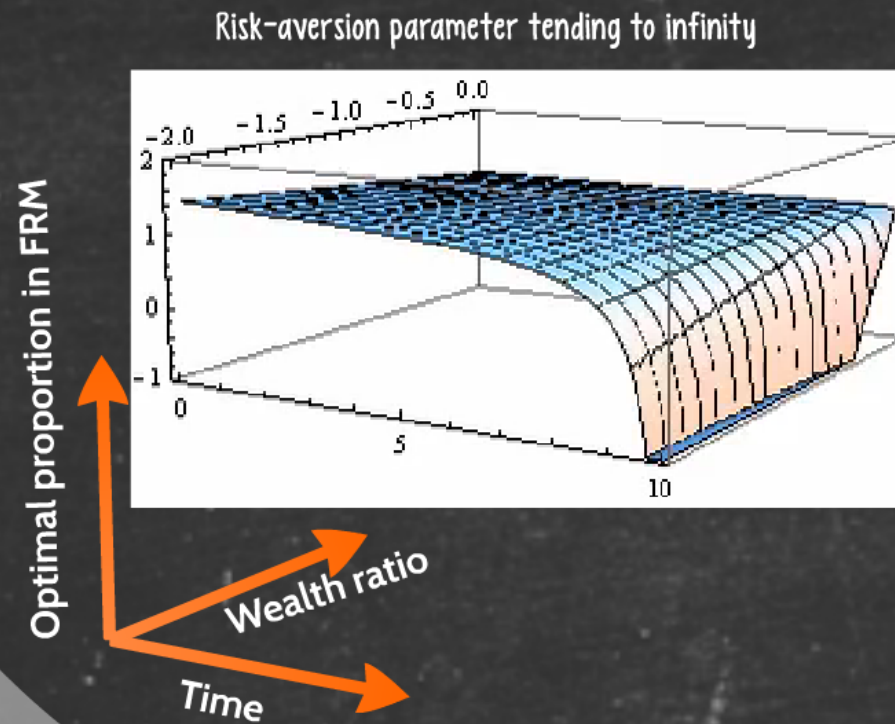
infinity



For closed-form solutions, Labor-income must behave "as if" it were a financial asset



The main advantage of the closed-form solutions, lies in the nice interpretation of the results



In conclusion: results on optimal mortgages can be found by including stochastic interest & labor income

- A Fixed Rate Mortgage (FRM), is found to be the optimal choice for an investor with constant Labor Income and infinite risk-aversion
- The Adjustable Rate Mortgage (ARM) with short-rate as interest, is found to be optimal only if the market price on interest risk is zero.

Further extension could be
owning a house as financial asset



If housesale is a 0-1 decision, the solution is a stopping time

Optimizing Expected Utility of Terminal Wealth:

$$V(t, x, h) = \sup_{\tau} E_{t,x}[G(\tau, X(\tau) + H(\tau))]$$

Variational inequalities to solve:

$$V_t + \alpha h V_h + \frac{1}{2} \sigma^2 h^2 V_{hh} \leq 0$$

Cannot be solved
analytically on finite Horizon

$$V(t, x, h) \geq G(t, x + h)$$

$$\tau^* = \inf\{t : V(t, X(t), H(t)) = G(t, X(t) + H(t))\}$$

When the market is B/S-like:

$$dX(t) = rX(t)dt$$

$$dH(t) = \alpha H(t)dt + \sigma H(t)dW(t)$$

In conclusion: Even simple extensions cannot be solved analytically & must be attacked by numerical methods

Numerical Methods:

- Discretizing the problem and solve w/ stochastic programming
- Monte-Carlo Methods
- Finite difference discretization of the continuous set-up
- Finite Element Method in the continuous set-up

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