Collateral flows, funding costs, and counterparty-risk-neutral swap rates

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BASED ON JOINT WORKS WITH

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Agenda

1 Overview

2 Consistent valuation of swaps

3 Equilibrium swap rates

4 Cost of collateralization: case study

5 Conclusion

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Overview

Global financial crisis

- Counterparty risk and counterparty risk mitigation tools matter
  - ⋆ collateral rules and funding costs integral part of the transaction
  - ⋆ implications for pricing, hedging, market-to-market accounting
- Multicurve modelling
  - ⋆ LIBOR, EURIBOR, EONIA, EUREPO?

New regulation (Dodd-Frank, EMIR)

- clearing, netting, collateralization
- collateral quality, segregation, re-hypothecation
- replacement cost, close-out conventions

Valuation challenges (e.g., Brigo's Counterparty Risk FAQ, Nov 2011)

- Credit/Debit Valuation Adjustment (CVA/DVA)

Questions

Consistent valuation with counterparty risk and liquidity risk

- Swap rates endogenize collateral flows and funding/opportunity costs
- Root finding, stochastic approximation algorithms.

Impact of different collateral rules / conventions

- Partial vs. full collateralization
- Symmetric vs. asymmetric collateral rules
- Segregation vs. rehypothecation

Quantifying the cost of collateralization

- Benchmark: interest-rate swaps (IRS) market
- Case study: bespoke longevity swaps

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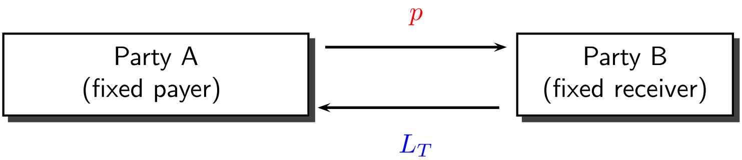
Common pitfalls

Interest-rate swaps (IRS)

- almost every IRS bilaterally collateralized
- cash collateral in over 90% of the cases



Common pitfalls

Interest-rate swaps (IRS)

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Duffie/Singleton valuation formula:

- unitary notional, single payment
- LIBOR default spread, λ

$$V_0 = E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \lambda_t)\mathrm{d}t\right)(L_T - p)\right]$$

- Exceptions: He (2001) and Collin-Dufresne/Solnik (2001) set $\lambda = 0$.

Bilateral default risk

Allow for credit quality of counterparties (Duffie/Huang, 1997)

- party A pays fixed, party B pays floating
- default intensities λ_t^A, λ_t^B, and recovery rates R^A, R^B
- fixed payer's viewpoint

$$V_0 = E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \Lambda_t)\mathrm{d}t\right)\left(L_T - p^d\right)\right]$$

$$\Lambda_t := \begin{cases}(1-R^A)\lambda_t^A & \text{if } V_t < 0 \\ (1-R^B)\lambda_t^B & \text{if } V_t \geq 0\end{cases}$$

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- full collateralization, $R^A = R^B = 1$

$$V_0 = E^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t\mathrm{d}t\right)(L_T - p^d)\right]$$

...default-free, risk-neutral valuation formula...

Collateralization

Collateral fractions $(c_t^h)_{t\geq 0}$ (ITM), $(c_t^p)_{t\geq 0}$ (OTM) [hedger's viewpoint]

- $c_t^h V_{t-} 1_{\{V_{t-}>0\}}$ cash held, $c_t^p V_{t-} 1_{\{V_{t-}<0\}}$ cash posted
- funding cost / opportunity cost / capital relief
- δ_t^h net gain from holding collateral (r rebated)
- δ_t^p net cost of posting collateral (r rebated)

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- swap's market value

$$V_0 = E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \Gamma_t)\mathrm{d}t\right)(S_T - p^c)\right]$$

$$\Gamma_t := \begin{cases} (1-c_t^p)^+ \lambda_t^A - \delta_t^p c_t^p & \text{if } V_t < 0 \\ (1-c_t^h)^+ \lambda_t^B - \delta_t^h c_t^h & \text{if } V_t \geq 0 \end{cases}$$

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- Full collateralization ($c^{p,h} = 1$), symmetric costs/spreads (δ, λ):

$$r_t + \Gamma_t = r_t - \delta_t$$

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Swap rates

Swap rate p^c from $V_0 = 0$

$$p^c = E^{\mathbb{Q}}[L_T] + \frac{\mathsf{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T (r_t + \Gamma_t)\mathsf{d}t\right), L_T\right)}{E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \Gamma_t)\mathsf{d}t\right)\right]}$$

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Root finding ($V_0(p^c) = 0$) and stochastic approximations

- Robbins-Monro, Polyak-Ruppert averaging
- Main issue is unbiased estimator of $V_0(p)$ when using American MC

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Collateral rule examples

- collateral thresholds based on credit ratings, CDS spreads, etc.
- $c_t^p = c_t^h = 1$ (full collateralization)
- $c_t^p = \alpha$, $c_t^h = \beta$, with $\alpha, \beta \in [0, 1]$
- $c_t^p = 1_{\{L_t \leq \beta(t)\}}$, $c_t^h = 1_{\{L_t \geq \alpha(t)\} \cup \{\lambda_t^B \geq \gamma\}}$, with $\alpha(\cdot) > \beta(\cdot)$, $\gamma \geq 0$

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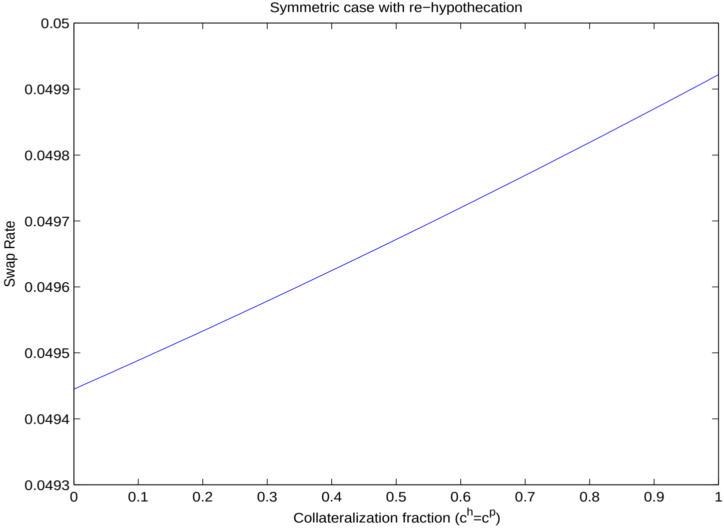
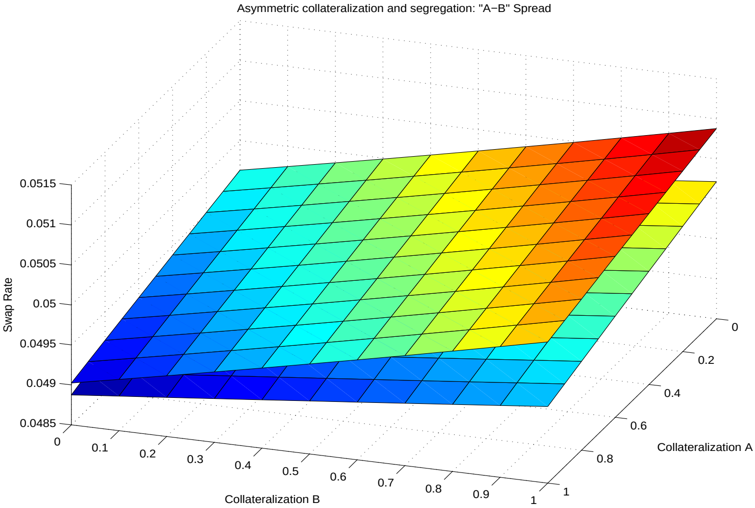
- $c_t^p = 1_{\{V_t(p^c) \le \underline{v}\}}$ and $c_t^h = 1_{\{V_t(p^c) \ge \overline{v}\}}$, with $\underline{v} < \overline{v}$

Symmetric collateralization with re-hypothecation



(A)symmetric collateralization with segregation



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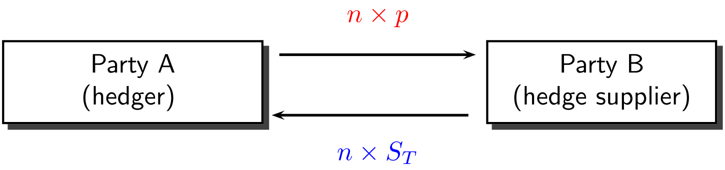
Longevity swaps

Date	Hedger	Size	Term (yrs)	Type	Interm./supplier
Jan 08	Lucida	Not disclosed	10	indexed	JPM ILS funds
Jul 2008	Canada Life	GBP 500m	40	bespoke	JPM ILS funds
Feb 2009	Abbey Life	GBP 1.5bn	run-off	bespoke	DB ILS funds Partner Re
Mar 2009	Aviva	GBP 475m	10	bespoke	RBS
Jun 2009	Babcock International	GBP 750m	50	bespoke	Credit Suisse Pacific Life Re
Jul 2009	RSA	GBP 1.9bn	run-off	bespoke	GS (Rothesay Life)
Dec 2009	Berkshire Council	GBP 750m	run-off	bespoke	Swiss Re
Feb 2010	BMW UK	GBP 3bn	run-off	bespoke	DB Paternoster
Dec 2010	Swiss Re (Kortis bond)	USD 50m	8	indexed	ILS funds
Feb 2011	Pall Pension Fund	GBP 70m	10	indexed	JPM

Bespoke longevity swaps

Stylized example: single payment at time $T > 0$

- notional $n > 0$, fixed payment $p \in (0, 1)$
- variable payment S_T (realized survival rate)



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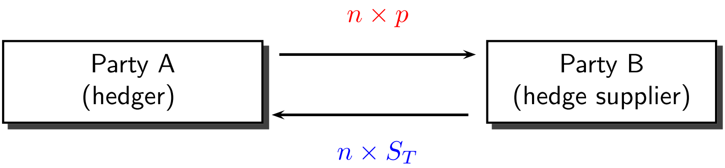

Swap value (hedger's viewpoint)

$$V_0 = nE^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \mathrm{d}t\right)(S_T - p)\right]$$

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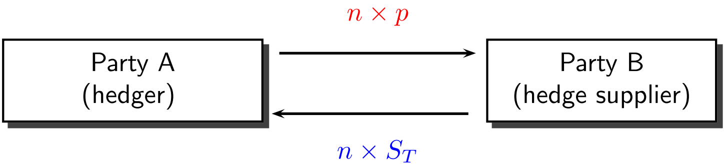

Longevity swap rate

$$p = E^{\mathbb{Q}}\left[S_T\right] + \frac{\text{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T r_t \text{d}t\right), S_T\right)}{E^{\mathbb{Q}}\left[\exp\left(-\int_0^T r_t \text{d}t\right)\right]}$$

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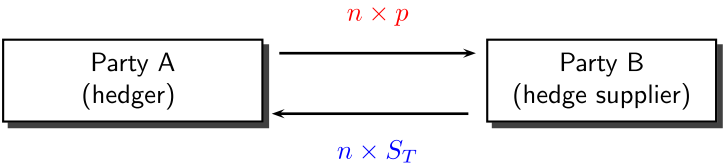

Longevity swap rate (r, S uncorrelated)

$$p = E^{\mathbb{Q}} [S_T] + 0$$

Bespoke longevity swaps

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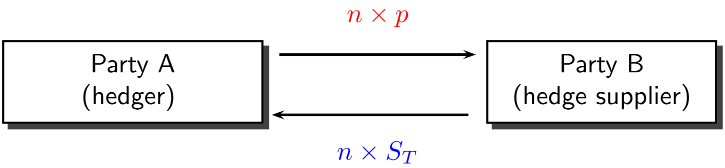
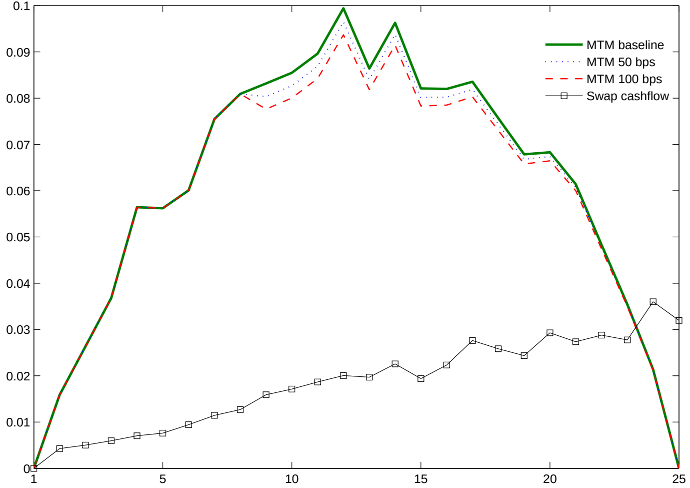


Longevity swap rate (r, S uncorrelated)

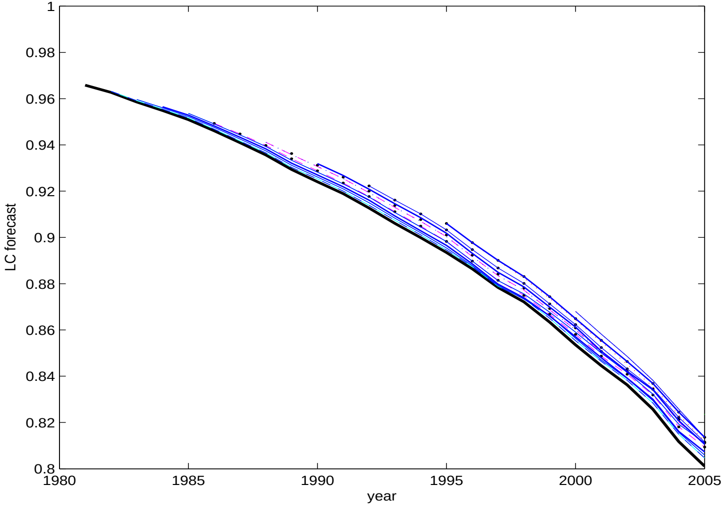
$$p = E^{\mathbb{Q}}[S_T] + 0$$

Useful baseline case $p = E^{\mathbb{P}}[S_T]$ (best estimate).

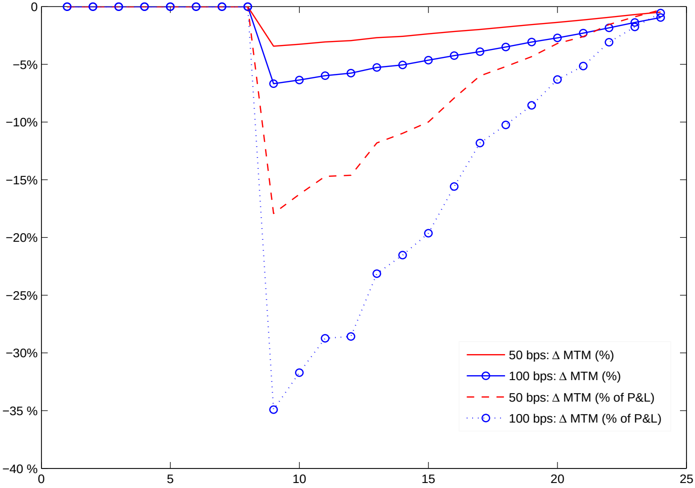
Cashflows and marking-to-market



Longevity swap rates



Hedge supplier's credit deterioration



Fully fledged calibration

Building blocks

- two-factor short rate model
- TED spread for λ^B
- $\lambda^A = \lambda^B + \Delta$, $\Delta > 0$
- net cost of collateral in IRS market (calibration of Johannes/Sundaresan, 2007)
- Lee-Carter mortality model

Two approaches to collateral costs δ^h, δ^p

⋆ funding costs associated with collateral flows

⋆⋆ simulate Solvency II capital charges (1-year 99.5% VaR) accruing from representative longevity-linked liability;
opportunity cost of (say) $6\% + LIBOR$ incurred on capital charges

Fully fledged calibration

$\mathbb{Q}$-dynamics of state variable process X

$$
\begin{aligned}
\mathrm{d}X_t^1 &= \left(k_1(X_t^2 - X_t^1) - \eta^1\right)\mathrm{d}t + \sigma_1 \mathrm{d}W_t^1 \\
\mathrm{d}X_t^2 &= \left(k_2(\theta_2 - X_t^2) - \eta^2\right)\mathrm{d}t + \sigma_2 \mathrm{d}W_t^2 \\
\mathrm{d}X_t^3 &= \left(\kappa_3(\theta_3 - X_t^3) + \kappa_{3,1}(X_t^1 - \theta_2) + \kappa_{3,4}(X_t^4 - \theta_4) - \eta_3\right)\mathrm{d}t + \sigma_3 \mathrm{d}W_t^3 \\
\mathrm{d}X_t^4 &= \left(\kappa_4(\theta_4 - X_t^4) + \kappa_{4,1}(X_t^1 - \theta_2) + \kappa_{4,2}(X_t^2 - \theta_2) - \eta_4\right)\mathrm{d}t + \sigma_4 \mathrm{d}W_t^4 \\
\mathrm{d}X_t^5 &= \left(\kappa_5(\theta_5 - X_t^5) + \kappa_{5,1}(X_t^1 - \theta_2) + \kappa_{5,2}(X_t^2 - \theta_2) + \kappa_{5,3}(X_t^3 - \theta_3)\right. \\
&\quad \left. + \kappa_{5,4}(X_t^4 - \theta_4) + \kappa_{5,6}(X_t^6 - E_0[X_t^6]) - \eta_5\right)\mathrm{d}t + \sigma_5 \mathrm{d}W_t^5 \\
\mathrm{d}X_t^6 &= \left(A(t) + B(t)(X_t^6 - a(t))\right)\mathrm{d}t + \sigma_6(t)\mathrm{d}W_t^6
\end{aligned}
$$

- $r = X^1$, mean reverting to random target X^2
- $\lambda^B = X^3$, TED spread
- X^4, net cost of collateral in IRS markets (Johannes/Sundaresan, 2007)
- X^5 net cost of collateral for longevity risk exposures
- X^6 continuous time version of Lee-Carter model

Parameter estimates

Parameter estimates

- Treasury/IRS market: Johannes/Sundaresan (2007)
- Mortality: US/UK HMD data
- Net cost of collateral:

  i) $\delta^h = \delta^p = \lambda^A = X^{(3)} + \Delta$, $\Delta \in \{0, 0.01, 0.02\}$
  ii) $\delta^h = \delta^p = X^{(5)}$

κ_1	0.969	η_1	-0.053	σ_1	0.008	UK	
κ_2	0.832	η_3	-0.014	σ_2	0.155	δ_K	-0.888
κ_3	1.669	η_4	0.007	σ_3	0.009	σ_K	1.156
κ_4	0.045	η_5	0.055	σ_4	0.010	US	
κ_5	0.990	$\kappa_{5,1}$	0.147	σ_5	0.690	δ_K	-0.761
$\kappa_{3,1}$	-0.163	$\kappa_{5,2}$	1.340	θ_2	0.046	σ_K	1.078
$\kappa_{4,1}$	0.114	$\kappa_{5,3}$	2.509	θ_3	0.003		
$\kappa_{3,4}$	0.804	$\kappa_{5,4}$	-0.133	θ_4	0.007		
$\kappa_{4,2}$	-0.038	$\kappa_{5,6}$	-0.002	θ_5	0.115	$\rho_{1,2}$	-0.036

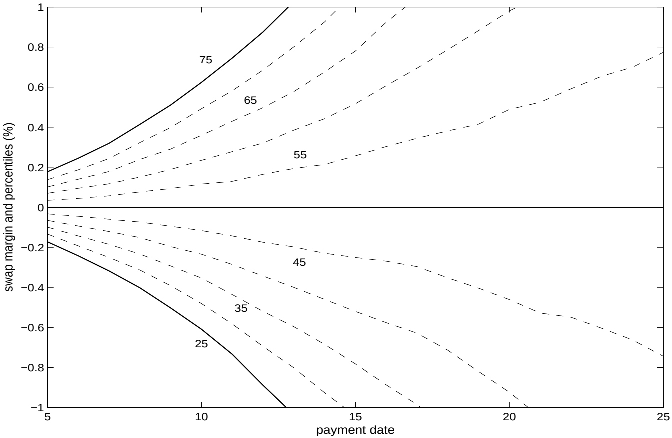
Longevity swap spreads

Underlying: 10,000 US males aged 65 at beginning of 2008. Term: 25 years.

- swap spreads (basis points), $p_T^c - E^{\mathbb{P}}[S_T]$:

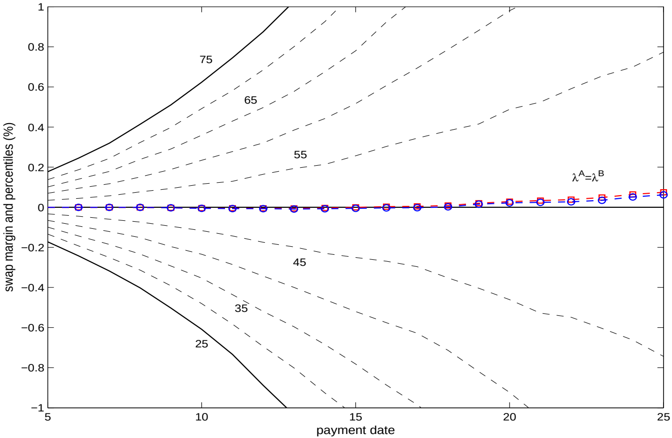
	Maturity payment (yrs)	$c^A = 0$ $c^B = 0$ (bps)	$c^A = 0$ $c^B = 1$ (bps)	$c^A = 1$ $c^B = 0$ (bps)	$c^A = 1$ $c^B = 1$ (bps)
$\lambda^{A,B} = \lambda$,	15	0.03	11.34	-11.76	0.05
$\delta^{A,B} = \delta$,	20	1.11	19.93	-17.94	0.86
$\delta = \lambda$	25	1.50	21.25	-18.35	1.24
$\lambda^A = \lambda^B + \Delta$,	15	5.45	16.79	-17.29	-5.84
$\delta^i = \lambda^i$,	20	10.16	28.95	-27.08	-8.23
$\Delta = 100$ bps	25	10.96	30.75	-27.76	-9.19
$\lambda^A = \lambda^B + \Delta$,	15	11.30	22.29	-22.90	-11.25
$\delta^i = \lambda^i$,	20	19.26	38.06	-36.16	-17.42
$\Delta = 200$ bps	25	19.46	40.27	-37.02	-18.38

Longevity swap margins



Funding costs case. Swap margins $\frac{p^c}{E^{\mathbb{P}}[S_T]} - 1$ against Lee-Carter mortality improvements quantiles for $\Delta = 0$ (dashed), $\Delta = 100$ bps (solid): no collateral (squares), full collateralization (circles).

Longevity swap margins



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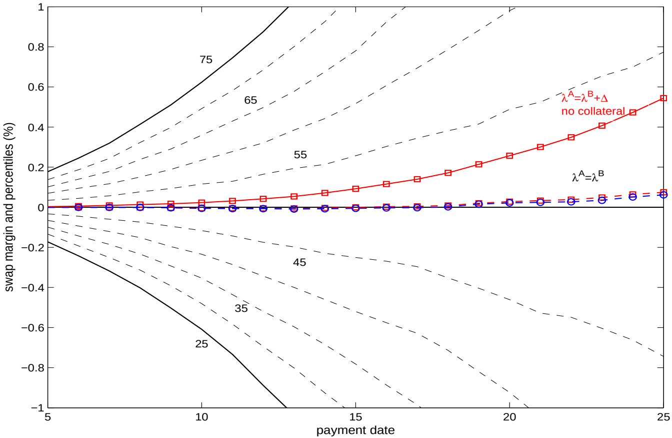

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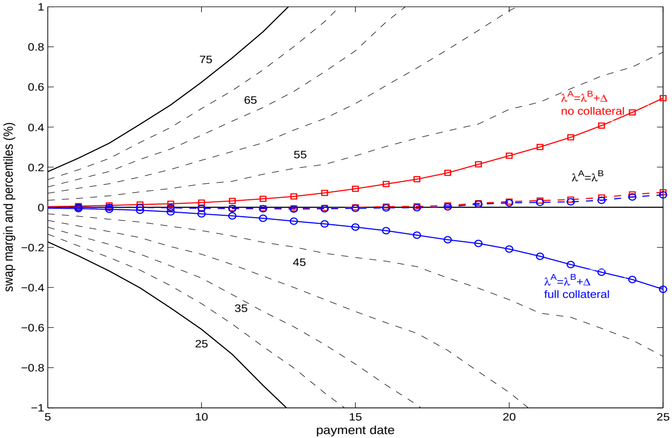

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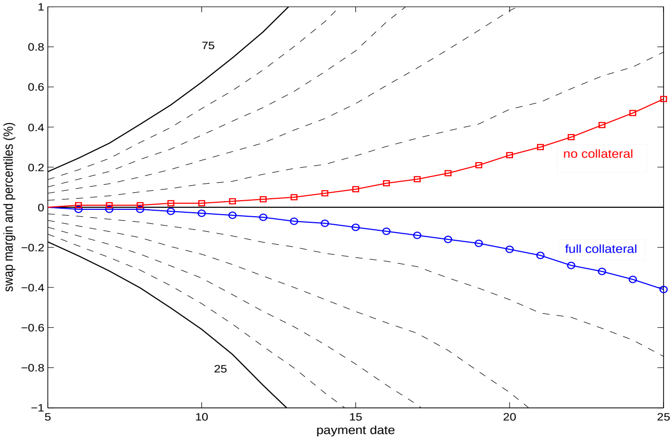
One-sided vs. two-sided collateralization



Funding costs case. Swap margins $\frac{p^c}{E^{\mathbb{P}}[S_T]} - 1$ against Lee-Carter mortality improvements quantiles. $\Delta = 100$ bps. No collateral (squares) vs. full collateralization: two-sided (circles), one-sided A (stars), one-sided B (diamonds).

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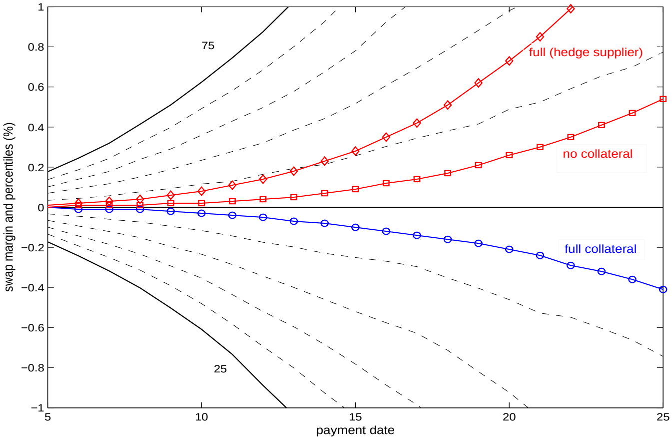

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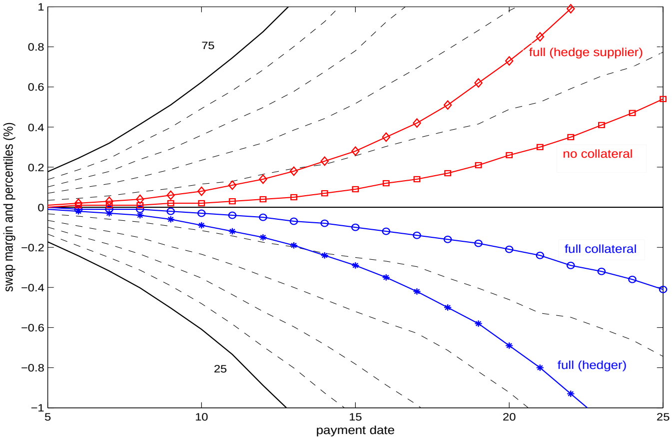

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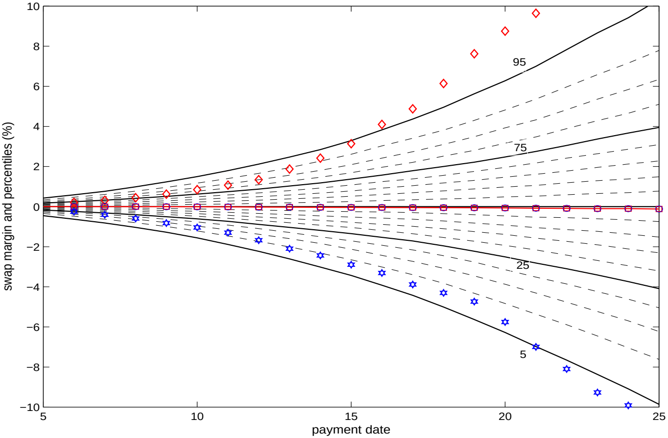
Capital charges approach



Opportunity cost case. Swap margins $p^c_{T_i}/p_{T_i} - 1$ against Lee-Carter mortality improvements quantiles for $\Delta = 0$: no collateral (squares), two-sided full collateralization (circles), one-sided A (stars), one-sided B (diamonds).

Understanding longevity swap rates

Two effects at play here

- longevity risk
- interest rate risk

$$p^c = E^{\mathbb{Q}}[S_T] + \frac{\mathsf{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T (r_t + \Gamma_t)\mathsf{d}t\right), S_T\right)}{E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \Gamma_t)\mathsf{d}t\right)\right]}$$

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Intuition

- A receives collateral when S_T is high, liability more capital intensive
- A posts collateral when S_T is low, liability less capital intensive

Understanding longevity swap rates

Two effects at play here

- longevity risk
- interest rate risk

$$p^c = E^{\mathbb{Q}}[S_T] + \frac{\text{Cov}^{\mathbb{Q}}\left(\exp\left(-\int_0^T (r_t + \Gamma_t)\mathrm{d}t\right), S_T\right)}{E^{\mathbb{Q}}\left[\exp\left(-\int_0^T (r_t + \Gamma_t)\mathrm{d}t\right)\right]} \quad \downarrow\downarrow$$

Intuition

- If A is ITM, collateral higher in low interest rate environments
- If A is OTM, collateral lower in higher interest rate environments

Comparison with IRS market

IRS spreads: difference betweeen futures price ($\delta = r$) and swap rate of collateralized IRS of corresponding maturity

	Maturity payment (yrs)	IRS $c^A = 0$ $c^B = 1$ (bps)	IRS $c^A = 1$ $c^B = 0$ (bps)	IRS $c^A = 1$ $c^B = 1$ (bps)	longevity $c^A = 0$ $c^B = 1$ (bps)	longevity $c^A = 1$ $c^B = 0$ (bps)	longevity $c^A = 1$ $c^B = 1$ (bps)
$\lambda^{A,B} = \lambda$,	15	-7.96	-44.97	-52.86	11.34	-11.76	0.05
$\delta^{A,B} = \delta$,	20	-12.68	-42.64	-56.22	19.93	-17.94	0.86
$\delta = \lambda$	25	-17.94	-40.98	-58.92	21.25	-18.35	1.24
$\lambda^A = \lambda^B + \Delta$,	15	-8.00	-67.87	-75.23	16.79	-17.29	-5.84
$\delta^i = \lambda^i$,	20	-12.65	-63.84	-77.42	28.95	-27.08	-8.23
$\Delta = 100$ bps	25	-17.65	-60.63	-77.64	30.75	-27.76	-9.19

Agenda

1 Overview

2 Consistent valuation of swaps

3 Equilibrium swap rates

4 Cost of collateralization: case study

5 Conclusion

Conclusion

Swap valuation with counterparty risk and liquidity risk

- Swap rates endogenize collateral flows generated by MTM procedure and associated funding/opportunity costs
- Root finding and stochastic approximation algorithms
- Even standard collateral rules may pose significant challenges

Impact of collateral rules / conventions

- Partial vs. full collateralization
- Symmetric vs. asymmetric collateral rules
- Segregation vs. rehypothecation
- Funding costs vs. opportunity costs

Quantifying the cost of collateralization

- The case of IRSs and bespoke longevity swaps
- Sign and magnitude of costs are far from obvious

THANK YOU

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